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DEPENDENCE OF FORMATION TIME OF OSCILLATING VORTICES ON EXCITATION FREQUENCY OF A ROTATING FLUID

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It was shown experimentally in [1-4] that free inertial oscillations excited in one or another way in a rotating fluid generate oscillating vortices in it, the structure and behavior of the vortices, as well as the whole structure of resulting flow, being determined by the set of eigenmodes dominating in the flow. But these characteristics are not related uniquely to a certain method of initiating the oscillations, since one and the same mode can be excited in many different ways.

Generators of periodic disturbances with frequencies corresponding to the eigenfrequencies were used in [1-3] to induce free oscillations with oscillating vortices. In [4] the oscillations were formed due to rapid disturbance of a rotating fluid from the equilibrium conditions by pulling a body through it. Despite the difference in the techniques applied in [1-4] to excite the oscillations, one could observe similar flow patterns with concentrated vortices during excitation of modes of similar geometry. Consequently, the effect of formation of oscillating vortices in itself is not related to a certain method of action on the rotating fluid, but is inherent in the rotating fluid itself. To a certain extent, this is its universal response to various disturbances.

Various inertial modes can generate vortices of different structure and oscillation frequency. But as has been shown experimentally [1-4], their formation does not depend directly on the mode geometry. On the other hand, it is obvious that the oscillating vortices do not appear in the absence of the disturbance generators and the oscillations due to them. Varying the parameters of initial flow and disturbance, one can attempt to find experimentally the generalized criteria responsible for the presence and absence of concentrated oscillating vortices in a rotating fluid. This paper presents a logical continuation of the work started in [1].

The experiments were performed using the same experimental device and measuring technique as in [1]. A schematic diagram of the device is presented in Fig. 1. A water-filled and hermetically sealed transparent cylindrical container 1 is mounted vertically on a rotating platform coaxial with the container. The bottom of the container 2 is made of elastic rubber. Periodic disturbance of the rotating liquid is provided by a generator consisting of hemispheres 3 mounted on a disc 4. The generator is supported on a shaft 5 coaxial with the container and which can rotate relative to the container and can move in the vertical direction.

Before the experiment the liquid in the container was brought into rigid rotation with angular velocity Ω, and the generator was driven up to angular velocity ω. Then the generator was raised through a height h from the position where the hemispheres touch the rubber membrane bottom. The hemisphere therefore pressed into the bottom and produced rounded bulges on it, which move with respect to the container with the required velocity. The flow induced by the disturbance was studied by visual observation. Polymer spheres of small negative buoyancy (ρ = 1.00-1.05 g/cm^3) up to 1 mm in size were used to make the flow visual.

Considering the disturbance technique and digressing from an exact description of the shape of the elastic bottom, one can characterize the problem using eight parameters. These are Ω, ω, h, H, and R (the height and radius of the container), ν (the kinematic coefficient of viscosity), and M and b (the number of disturbing bodies and radius of their arrangement, respectively). It is accepted that when $M > 1$ all bodies are placed at equal distance from the axis. Let us apply the same set of six independent dimensionless parameters as that used in [1], which involves H/R, b/R, h/R, M, and the relative excitation frequency $f = 1-\omega/\Omega$. We shall also apply the wave Reynolds number $\mathrm{Re} = f\Omega bMh\lambda/(2R)$, whose substantial effect on the flow was considered in [1]. The combination $f\Omega bMh/R$ involved in Re characterizes the relative velocity introduced by the

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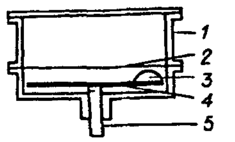

Fig. 1

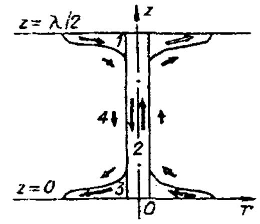

Fig. 2

disturbance, while λ/2 (the half wavelength along the rotation axis of the analogous length for the radial direction, if it is less than the latter) is the characteristic dimension with which the change of velocity in the wave occurs. The following parameters were held constant in the experiments: R = 25 cm, b/R = 0.78, the curvature radius of the hemispheres was 4.5 cm.

Since the rotating liquid is the elastic medium, the above excitation (at least, when its frequency is within the inertia range) should lead (and actually led) in the experiments to the excitation of inertial waves in the liquid. For the observed nonaxisymmetric waves in the cylindrical container with rigidly rotating liquid the linear theory gives [5].

$$v = I_m(\alpha_{mj} r/R)\exp(ik_{mj} z/H + im(\theta - \omega t));$$
$$k_{mj} = \alpha_{mj}\frac{H}{R}\left[\frac{4}{m^2 f^2} - 1\right]^{-1/2}. \tag{1}$$

Here r, θ, and z are the radial, angular, and longitudinal components of the cylindrical coordinate system, v is the z-component of the velocity, ω is the wave frequency, m = 1, 2, ... is the angular wave number, I_m is a Bessel function of the first kind of order of m, and j is an integer having the meaning of the number of zeros of radial velocity component over the interval $0 < r \leq R$; $f = 1 - \omega/\Omega$. The nonpenetration condition with r = R leads to the equation for $\alpha = \alpha_{mj}$

$$2I_m(\alpha) + f\alpha\frac{dI_m(\alpha)}{d\alpha} = 0,$$

which together with (1) determines the dependence of the longitudinal wave numbers k_{mj} on the dimensionless relative frequency f. With $|f| > 2/m$ the numbers k_{mj} are purely imaginary.

A disturbance with an arbitrary relative frequency f induces travelling waves with different k_{mj}, which, due to repeated reflections, interaction with each other, and dissipation should yield in the simplest case a picture of steady forced oscillations. If even for one of the waves the relationship $k_{mj} = n\pi$ (n = 1, 2, ...) holds to a sufficient accuracy, then the conditions of resonance excitation of the appropriate mode of free oscillations are realized in the container. The aggregate of three integers (m, j, n) prescribes the eigenmode configuration.

Substantial excitation of any inertial mode in the general case produces a system of several cyclone and anticyclone vortices [1-4]. The oscillating cyclone vortices are of high radial, axial, and circular velocities of the fluid motion. The angular rotation velocity inside the cyclones can achieve 50Ω [2, 3]. Anticyclones do not acquire high velocities and cannot be found visually. Therefore, we will be further concerned only with cyclone vortices.

All cyclones described in [1-4] and the present work are of similar structure and behavior. We shall explain this similarity in detail.

The vortex appearing in the near-axial region under excitation of the axisymmetrical (m = 0) mode with one half-wave along the rotation axis (n = 1) can serve as the most simple, apparent, and easy-to-observe example of the cyclone. Such modes were observed in [2-4].

A diagram of the flow in the vortex is shown in Fig. 2. One can distinguish four characteristic zones in the flow: 1 and 3 are the annular regions of high radial velocity, 2 is the region of high axial velocity and high relative angular velocity or rotation, and 4 is the region of moderate velocities. At one half-period of the vortex oscillations the liquid flows from region 1 to region 2, and then from region 2 to region 3. In this case, as the circulation is retained, the rotation velocity of the liquid flowing from region 1 to region 2 increases and the maximum angular velocity of rotation is attained to the moment of stoppage

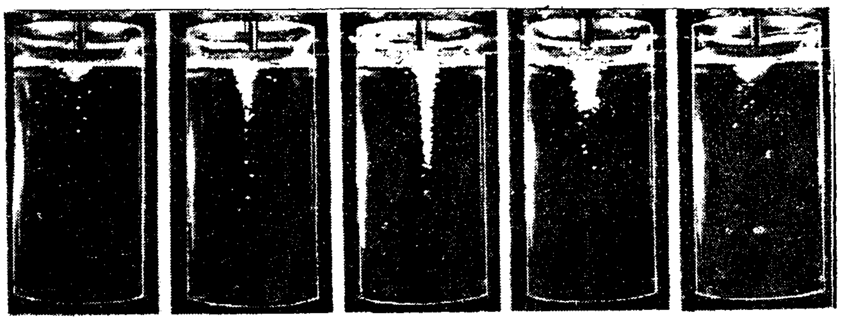

Fig. 3

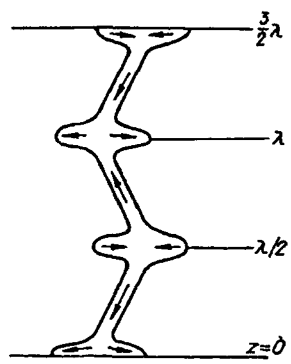

Fig. 4

or radial flow at the vortex axis at the level z = λ/2. For the same reason when the liquid flows from the region 2 to region 3 its rotation velocity decreases, and the minimum angular velocity of rotation is achieved up to the moment of stoppage of radial flow at the annular region at z = 0. At the other half-period of the vortex oscillation the flow pattern changes inversely with time.

In any oscillation phase the direction of the relative rotation of the liquid remains cyclonic in the greater part of the region 2 (except for the vicinities of the levels z = 0, λ/2). Figure 3 shows a series of frames of one period of the vortex oscillations obtained in [4], which illustrates the diagram of the flow from Fig. 2. The flow was made visual with polyethylene granules (ρ = 0.95 g/cm^3).

The flow shown in Fig. 2 is the minimum structure element of any one-mode oscillating cyclone. The diagram of the flow in the axial vortex during excitation of the mode (0, j, n) is constructed by vertical matching of n coaxial half-wave sections analogous to that shown in Fig. 2. (The velocity fields in the neighboring sections, being in antiphase to each other, provide discontinuity.) Thus, with n > 1 the regions of radial convergence–divergence are placed not only at the upper and lower boundaries, but also inside the liquid.

The one-mode vortex placed outside of the container axis is roughly of the same structure as the axial vortex with the same n, but it has its peculiar features stemming from the surrounding flow. Being in the inertial wave, the vortex is apt to drift, which results in the incidence and precession of its axis to the wave frequency. Depending on the amplitude and geometry of the wave, the slope of the axis can be either hardly noticeable or rather substantial. In the adjacent vertical sections the drift of the vortex is accomplished again in antiphase; therefore, its axis resembles a "snake." The qualitative structure of the off-axis one-mode cyclone is shown in Fig. 4 by the example of the vortex for the mode with n = 3.

The one-mode vortices with equal n are of the same qualitative structure irrespective of m and j and in the general case are distinguished by the oscillation frequency, the slope of the axis, and other qualitative characteristics of the flow.

Under arbitrary disturbance of the rotating liquid, for example, as in [4], the set of inertial eigenmodes is excited simultaneously. The oscillating vortices appear in this case as well, but they do not have to be of such an explicit periodicity in space and time. Therefore, it is much more difficult to indicate the modes responsible for the vortex formation. However, it is possible with one or two dominating modes. The two-mode vortex is subject to two-period oscillations (beats), and s, having determined both periods, one can reveal the frequencies of the generating vortices, and taking account of the evolution

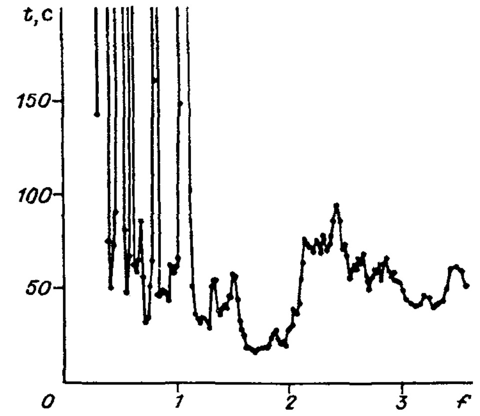

Fig. 5

in the vertical vortex structure, determine the modes themselves. This can occur when two modes with different m and j dominate each at its own radius range, and the vortices placed in these annular zones will follow mainly the frequency and structure of one of the modes.

Introducing disturbances into a rotating fluid, one can obtain a wide variety of flow patterns. Depending on the amplitude and geometry of the excited modes, as well as on the stage of evolution of the flow, there can be different numbers of concentrated oscillating vortices. However, according to the purpose of the present paper only the presence or absence of the vortices is of interest for us, while their number, vertical structure, intensity, and location can be neglected. Therefore, below we discuss the most general and substantial characteristics and peculiarities of the flow and disturbance.

With strict formulation the question of presence or absence of concentrated oscillating vortices in a rotating liquid is converted into that of appearance or absence of such vortices over the given time interval due to the action of the given disturbances generator on an originally equilibrium rotating liquid. That is the reason why the time of generation of only one vortex in the flow was the principal quantity measured in the experiments. Since there are no examples of more or less accurate definition of this parameter for oscillating vortex in the literature, the author had to select the time interval to be measured to ensure high reproducibility and minimum random error in the measurements.

For this purpose, we distinguished in numerous observations the most remarkable and reliably recorded phase of the vortex evolution. The time of inception of the phase from the moment the disturbance was introduced was recorded using the stopwatch. The most characteristic phase in the vortex manifested itself for the first time as a local twisted jet with subsequent clearly pronounced reverse motion. This definition of the time moment is rather qualitative, since it does not involve any quantitative characteristic. However, the disturbance of a rotating fluid produces vortices different in size, structure, intensity, and oscillation frequency. It is difficult to establish a simple unified quantitative criterion without losing the qualitative similarity.

A reasonable approach may consist in constructing a dimensionless parameter on the basis of the vortex characteristics and measuring the time it takes for the parameter to achieve the given value. However, in this case it is necessary to find a compromise between the strictness of the parameter definition and the technological measuring capabilities which are rather scanty in the given case. We do not know in advance where the first concentrated vortex will appear under an arbitrary disturbance of the rotating liquid; moreover, its location can vary. In these conditions any instrumental technique for measuring the local flow characteristics is unsuitable. The only way is to measure the velocity field. But the flows realized in the tests are strongly nonuniform and unsteady and the available techniques for measuring the velocity fields are either unsuitable or very expensive and labor consuming, and therefore, of low efficiency. The compromise way out is visual observations.

Since it is difficult to perform any quantitative measurements of the vortex parameters in the process of visual observations, in order to register the moment of inception of the oscillating vortex we used the above qualitative definition. However, on the basis of the observations, one can try to give its quantitative interpretation. Actually, the presence of the axial vortex jet can be reliably established only if the axial shift of buoyant particles in the vortex over the half-period of its oscillation exceeds the transverse size of the vortex. Although there is a certain discrepancy in the sizes of the vortices observed in different tests, the threshold value of the shift can be specified approximately as 1 cm.

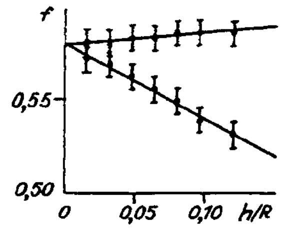

Fig. 6

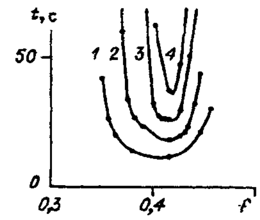

Fig. 7

In the great majority of cases, the approximate nature of the threshold value did not influence the measured inception time and the measurement accuracy, since the vortex passes the critical phase of its evolution very quickly, and the time measurement error depends on the behavior of the oscillating vortex, and, in the best case, is equal to one period of its oscillations. In actuality the qualitative character and roughness of the definition applied for the moment of inception of the vortex manifest themselves only with a slow rate of vortex evolution, which results in an increase in the absolute measurement error. But since in this case the duration of the vortex formation is also great, this does not affect significantly the relative measurement error. Thus, the successful selection of the moment for the measurements makes it possible to avoid extremely serious technical difficulties.

Having discussed the measuring technique, let us consider the essence of the work. The duration of the formation process of oscillating vortices in the resonance excitation of eigenmodes was studied in [1]. It was shown that the resonance excitation of the inertial wave can be accomplished by a periodic disturbance only when a certain critical wave Reynolds number is exceeded. This condition leads in its turn to a limited set of eigenmodes capable of being excited. However, in practice the frequency of the periodic disturbance of a rotating liquid is often not equal to one of the frequencies of the modes which can be excited. In this connection, knowledge of the criticality of the vortex formation effect to the frequency of external periodic disturbance is of concern.

It has been reported in [1] that the time of formation of concentrated oscillating vortices strongly depends on the excitation frequency and has its local minimum value when the excitation frequency is equal to one of the eigenfrequencies. In this paper we present a more detailed study of this dependence and show that the equality of the excitation frequency to one of the eigenfrequencies of undisturbed flow is not a necessary condition for vortex formation, and the effect of formation of oscillating vortices falls in a rather broad disturbance frequency range.

Several different runs of experiments with a container of height $H = 18.4$ cm and invariable revolution period 3.00 $\pm$ 0.01 sec were performed to gain an idea of the typical character of the dependence of the vortex formation time on the disturbance frequency. The tests with the greatest number of measurements and the widest range of excitation frequency were held with the disturbance amplitude $h/R = 0.08$ and the most usual geometry, i.e., with $M = 1$. The results of this run are shown by the points in Fig. 5 (the relative disturbance frequency f is along the horizontal axis, and the time of vortex initiation is along the vertical axis). Linear interpolation of the results is carried out at the plot.

The experimental data on Fig. 5 are obtained by averaging over 3-10 measurements made in different tests under identical conditions. To calm the liquid and restore it into the initial state, a pause was made between the tests. During the pause the disturbance was removed. The minimum duration of the pause was 10 min, which for the given conditions is sufficient in most cases to provide good reproducibility of the measurement results irrespective of the concrete time interval between the tests. The measurements were carried out with the frequency increment 0.02 for $f \leq 3$; and 0.04 for $f > 3$. In this case the error for f was at least an order of magnitude less than the frequency increment in the measurements. The points at which the formation of concentrated vortices was not detected or took place after a long time are beyond the plot. The interpolated straight lines going upwards in Fig. 5 indicate their presence. If no vortices appeared over a time equal to the minimum pause between the tests (10 min), the test was interrupted.

From the results of Fig. 5 one can observe the presence of at least six frequency intervals wherein the vortex formation was not found. All of them are in the region of low relative frequencies. The approximate location of the intervals is as follows: [0, 0.28], [0.32; 0.38], [0.48; 0.52], [1.04; 1.06] and in the vicinity of the points 0.6 and 0.82. However, one cannot be certain that whatever f may be, there is no vortex formation within the above intervals, because it is not inconceivable that there can be very narrow dips of the resonance curve. Representative in this sense is the point with $f = 0.3$ (at the extreme left of the plot). No vortex formation to the left or to the right of the point was found. To determine reliably the number and boundaries of "empty" spaces, one should thoroughly study the inertial spectrum using the criterion found in [1] from the mode

Reynolds number and carry out an experimental check of the questionable points. The author omitted these operations, since they are labor consuming and are inconvenient for the goals of this research.

From the analysis of the plot in Fig. 5 one can easily notice that there are no "empty" spaces with great f, although when $f > 1$ there are no modes with $m > 1$ in the spectrum. To explain the difference, one should recall wave attenuation due to viscosity. The effect of viscosity on the excitation of the inertial wave is characterized by the wave Reynolds number $Re = f\Omega bMh\lambda/(2vR)$ introduced in [1]. The efficient excitation of the inertial wave is possible only if the Reynolds number for the given wave exceeds a critical value. Since the scale of the wave is a factor of the Reynolds number, with fixed excitation frequency f only a limited number of modes of larger scale and more convenient geometry among those of the frequencies close to f can be excited.

With low f the threshold scale for the waves is great; there are few modes capable of being excited, and they are often of considerably distinct frequency. For the given excitation frequency it may appear that there is no mode which can be excited nearby. In this case vortices do not originate, and the excitation due to the viscous attenuation of the inertial waves cannot disturb the flow. Hence the appearance of the "empty" space in the plot in Fig. 5 is clear. In the part of the range where these spaces occur, the dependence of the vortex formation time on the excitation frequency is of the type of the set of explicit resonance minima which can be created by one mode, as well as a series of modes.

The mode or modes that dominate in the formation of the vortices can be seen by observing the pattern of flow with vortices. In this case the number n is determined from the number of elementary vertical sections observed in the flow, while m is found as the number of vortex oscillations in one revolution with respect to the disturbance generator. Taking into account the number and arrangement of the vortices and having measured their oscillation frequency, one can also reproduce the wave number j, if the latter is not too large. Thus, the resonance minimum with $f = 0.40$ to 0.46 is generated by the mode (3, 1, 1); with $f = 0.54$ to 0.58, by the mode (3, 1, 2); with $f = 0.62$ to 0.80 by the modes (1, 4, 1) and (2, 1, 1) and with $f = 0.84$ to 1.02, by at least three modes: (1, 3, 1), (2, 1, 2), and (2, 1, 3). It is natural that with $M = 1$ not only the modes with $m = 1$ are excited, but the harmonics with high m as well. With higher disturbance frequencies the resonance minima are joined without leaving free spaces, which can be seen in Fig. 5.

Thus, in the range f in which the eigenfrequencies are present, the character of the dependence of the vortex formation time on the excitation frequency is clear. The dimensionless frequency of the liquid oscillations of the mode with eigenfrequency f_{mjn} and $m \neq 0$ equals mf_{mjn}. Under normal excitation of the mode with the help of M bulges, the number m should be divisible by M, while $f_{min} \approx f$, and consequently the frequency of oscillations is approximately divisible by Mf.

The experimental results indicate that the oscillating vortices are formed also with $f > 2$, where no eigenfrequencies are present. It appears that in this case the relationship $f_{min} \approx f$ does not hold and the mechanism of vortex formation is somewhat different. The vortices are formed not only due to the excitation of the mode with the oscillation frequency of the liquid close to Mf or divisible by it, but also due to the generation of a subharmonic. Mainly it is this mechanism that is responsible for the generation of the vortices in the whole subcritical excitation frequency range. In all cases up to $f = 3.56$ (the limit of the examined range) the mode was excited with half of the oscillation frequency as compared to Mf, while the wave geometry was determined by f. The relationship $mf_{mjn} \approx 0.5Mf$ was satisfied instead of the approximate equality f and f_{mjn}. At the least, subharmonics with three-different values of n, equal to 1, 2, and 3, were observed in the tests with different f.

The subharmonic can be generated in the experiment even with $f < 2$, although the supercriticality probably plays a role. Thus, the excitation of the subharmonic was observed in tests arranged in the same container with $H = 18.4$ cm, period of revolution $T = 6$ sec, number of bulges $M = 3$, disturbance amplitude $h/R = 0.12$, and relative frequency for the modes with $m > 1$; for the modes with $m = 1$ disturbances with $M = 3$ are inconvenient from the geometric viewpoint. Nevertheless, the resulting flow pattern was the typical standing inertial wave with concentrated vortices, the vortices being composed of three sections (which corresponds to $n = 3$) and oscillating synchronously to every other bulge encountered along the path.

Thus, the generation of the subharmonic with half oscillation frequency is obvious, and taking into account the values of f and n, one can determine the resulting mode with great probability. It appeared to be the mode (1, 3, 3). The last example shows clearly how peculiar the mechanism of formation of concentrated oscillating vortices in a rotating fluid can be in each particular case.

Now that the character of the influence of the disturbance frequency on the time of vortex inception is roughly understood, let us discuss the tendencies of the flow evolution observed in the tests. The fact that there are no "empty" spaces with great f in the spectral characteristic of the vortex formation (see Fig. 5) likely indicates that the modes capable of being excited are of close frequency and, with any disturbance frequency f, there is always at least one mode of suitable geometry and frequency, whose excitation is accompanied by the formation of concentrated oscillating vortices. But then the resulting pattern of the flow with vortices should display the features of this mode.

However, this is not always so in practice. The flow pattern commonly observed in the tests is that corresponding to another mode of rather distinct frequency. This is shown by the discrepancy between the obtained and expected numbers of vortex sections and the values of their oscillation frequency.

The deviation of the resonance frequency depending on the disturbance amplitude was also found, which can be illustrated by the example of the results presented in Fig. 6, where the location of the boundaries of the resonance zone in the frequency f is shown as a function of the disturbance amplitude h/R of the mode (3, 1, 1) in a container of height H = 8.6 cm with revolution period T = 2.99 sec. The calculated eigenfrequency for this mode is 0.579. In this case, as in [1], the resonance zone was defined as the increment of the excitation frequency f in the vicinity of the eigenfrequency, within which the time of vortex formation differs by no more than one period of vortex oscillations. Figure 6 shows the approximating straight lines plotted for each boundary with the least squares method.

The results in the plot point to the fact that with increase in h/R not only the width of the resonance zone increases linearly, which was mentioned in [1], but also f decreases linearly for the middle point of the resonance interval, which indicates the decrease of the effective eigenfrequency of the resonance mode. This effect is manifested in low containers. In higher containers (for example, of H = 18.4 cm) the effect was not observed. However, if we do not confine ourselves to the region of experimentally accurate resonance and consider the resonance dip as a whole for the mode, which was obtained with different h/R, somewhat similar results can be found for containers of greater height as well.

The above is illustrated in Fig. 7 where resonance curves for the formation time of the oscillating vortices are presented, which were obtained with different disturbance amplitudes for the mode (3, 1, 1) in a container of height H = 18.4 cm with revolution period T = 3 sec. The calculated eigenfrequency for this mode is 0.412. The experimental results are shown by the points; curves 1-4 represent a cubic spline interpolation and correspond to excitation amplitudes h/R equal to 0.12, 0.08, 0.048, and 0.032. It is noteworthy that with increase in the disturbance amplitude the resonance dip expands asymmetrically: the shift of the left wall is greater than that of the right one. This suggests again the decrease of the effective eigenfrequency of the resonance mode.

The experiments make it possible not only to reliably establish the above fact, but also to provide a reasonable explanation. In some of the experiments we observed steady flow regime and found, for example, that the excitation of the mode (3, 1, 3) can yield a steady flow with two- and one-section vortices. This suggests that the vortices in the wave have n equal to 2 or 1 and not to 3 as in the beginning. The conditions of the experiments were as follows: container height H = 18.4 cm and radius R = 25 cm. M = 3, h/R = 0.12, f = 0.632. The initial experimental conditions for various regimes of steady flow varied only in the revolution period of the container, which was 6 and 4 sec for the two-section and one-section vortices, respectively. It was noticed also that in the steady flow regime the liquid on the whole rotates slower than the container. The level of deceleration could be estimated in measuring the revolution period of the suspended particles beyond the vortices. In the first case it appeared to equal ~ 7 sec, and in the second case, ~ 6 sec. It is this deceleration that is likely to be the reason for the reduction of the frequency of free oscillations.

Actually, the frequency of rotation of the generator ω and, accordingly, only the initial value of $f = 1 - \omega/\Omega$ (which could be true in the case of invariant rotation frequency Ω of the liquid) were given and maintained in the experiments. But since Ω reduces with time, this led in the mentioned experiments to the reduction of the observed eigenfrequency of free oscillations (which was calculated for constant rotation frequency of the liquid), and to the reduction of the effective excitation frequency f which in the calculation based on a constant Ω was maintained equal to 0.632, which is higher than the eigenfrequency of the mode (3, 1, 3), which is 0.618. The calculation of f from the revolution periods of a liquid obtained experimentally gives in the first case 0.571, which almost coincides with the frequency of the mode (3, 1, 2) equal to 0.569; and in the second case, 0.448, which corresponds to the right slope of the resonance dip of the mode (3, 1, 1) with the eigenfrequency 0.412. Thus, the origin of two- and one-section vortices becomes clear.

It is also clear that there is a limit on general deceleration of rotation in laboratory conditions. The factor preventing further deceleration is viscous friction in the boundary layers, which is characterized not by the wave Reynolds number, but by the scale one, $Re_s = \Omega RH/\nu$. The higher the Re_s, the higher the relative deceleration that can be achieved, and more intensive and diversified evolution of the flow structure is possible. This was proved by the above experiments.

Through which mechanism does the substantial deceleration occur? Obviously, the mechanism cannot be trivially viscous, since the disturbance interacts with the liquid through an elastic membrane which rotates together with the liquid. Furthermore, as was shown above, in the experiments the effect of the viscosity is quite opposite. It should also be noted that the rotation was observed to decelerate even in the case where the vortices are weak and not numerous. Most likely, the deceleration is primarily due to the force of nonviscous resistance against the body (bulge) motion. The force can be rather substantial with the relative motion velocities available in the experiment [6].

It is obvious that the deceleration of the liquid can lead to a reduction in the effective excitation frequency $f = 1 - \omega/\Omega$ only with positive ω. With negative ω the effective frequency f will tend to "drift" to the side of higher (or even supercritical) values. According to the current state f either subharmonics or the usual oscillations with current f in excess of the initial value will be excited during the flow evolution. The direction of the flow evolution in the most crucial practical case with $\omega = 0$ is of extreme interest and is not clear in advance. For the majority of cases the problem cannot be easily solved.

Thus, at least two different mechanisms of formation of oscillating vortices have been found experimentally. The presence of nonviscous resistance with sufficiently high Reynolds numbers (both wave and scale ones) sooner or later creates conditions for generating the oscillating vortices in the basic flow during its evolution. This circumstance explains the absence of empty spaces in the frequency characteristic of the vortex formation time with high f.

Summing up, we can state that the only factor preventing the formation of concentrated vortices during long periodic disturbance of the rotating liquid found for the time being is viscosity. With high Reynolds numbers over the disturbance frequency range examined the appearance of oscillating vortices depends only on time. Consequently, for the greater part of rotating flows it makes sense to discuss the appearance or absence of concentrated vortices only within the given time interval from the starting moment of the disturbance.

Finally, the results obtained in this work have traditional application to atmospheric vortices. Except for vortices of the smallest scale and of less interest, all atmospheric vortices are characterized by high values of the scale Reynolds number, and with allowance for the present disturbances and the wave Reynolds number. Therefore, with a sufficient lifetime the existence of a fine vortex structure is the rule rather than the exception. The cascade formation mechanism observed for vortices of different scale in natural conditions seems rather reasonable from the standpoint of the results presented.

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